2022 Fall PHYS2310 電磁學（Electromagnetism) Final (double sides)

[Griffiths Chs. 4 – 7.1] 2023/01/10, 10:10am – 12:00am, 教師：張存續 Total score: 110

◇ Useful formulas

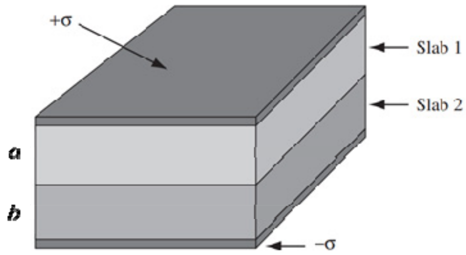
$$\mathbf{D}=\varepsilon_0\mathbf{E}+\mathbf{P}=\varepsilon_0(1+\chi_e)\mathbf{E}=\varepsilon\mathbf{E} \quad \text{and} \quad \mathbf{B}=\mu_0(\mathbf{H}+\mathbf{M})=\mu_0(1+\chi_m)\mathbf{H}=\mu\mathbf{H}$$

$$\nabla\cdot\mathbf{D}=\rho_f \qquad \nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0 \quad \nabla\cdot\mathbf{B}=0 \qquad \nabla\times\mathbf{H}-\frac{\partial\mathbf{D}}{\partial t}=\mathbf{J}_f$$

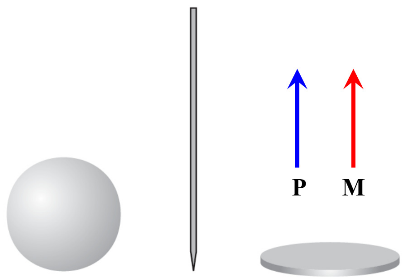
$$D_1^{\perp}-D_2^{\perp}=\sigma_f \quad \mathbf{E}_1^{//}-\mathbf{E}_2^{//}=0 \quad B_1^{\perp}-B_2^{\perp}=0 \quad \mathbf{H}_1^{//}-\mathbf{H}_2^{//}=(\mathbf{K}_f\times\hat{\mathbf{n}})$$

1. Explain the following terms. Write down the mathematical expressions *if applicable*. (6×5%)
   (a) Electric susceptibility (5%)
   (b) Ampere's law (5%)
   (c) The Coulomb gauge (5%)
   (d) Ohm's law with Drude model (5%)
   (e) Hysteresis loop and Curie temperature (5%)
   (f) Effect of a magnetic field on atomic orbits and diamagnetism (5%)

2. The space between the plates of a parallel-plate capacitor is filled with two slabs of linear dielectric material. The thicknesses of the slabs 1 and 2 are a and b respectively. Slab 1 has a dielectric constant of 9 and slab 2 has a dielectric constant of 4. The free charge density of the top plate is σ and on the bottom plate $-\sigma$.
   (a) Find the electric displacement **D**, the electric field **E**, and the polarization **P** in each slab. (5%)
   (b) Find the location and amount of all bound charges (ρ_b and σ_b). (5%)
   (c) Find the potential difference V between the metal plates. (5%)
   (d) Find the capacitance of the capacitor. (5%) [Hint: Consider the parallel-plate capacitor with the surface area A.]



2. Suppose the fields inside a large piece of material are $\mathbf{E}_0$ and $\mathbf{B}_0$, so that $\mathbf{D}_0=\varepsilon_0\mathbf{E}_0+\mathbf{P}$ and $\mathbf{B}_0=\mu_0(\mathbf{H}_0+\mathbf{M})$, where **P** and **M** are "frozen-in", so they don't change when the cavity is excavated.
   (a) Now a small spherical cavity (Fig. (a)) is hollowed out of the material. Find the field at the center of the cavity in terms of $\mathbf{E}_0$ and **P**. Also find the displacement at the center of the cavity in terms of $\mathbf{D}_0$ and **P**. (5%)
   (b) Do the same for a long needle-shaped cavity (Fig. (b)) running parallel to **P**. (5%)



(a) Sphere (b) Needle (c) Wafer

(c) Now a small spherical cavity (Fig. (a)) is hollowed out of the material. Find the field at the center of the cavity, in terms of $\mathbf{B}_0$ and **M**. Also find **H** at the center of the cavity, in terms of $\mathbf{H}_0$ and **M**. (5%)

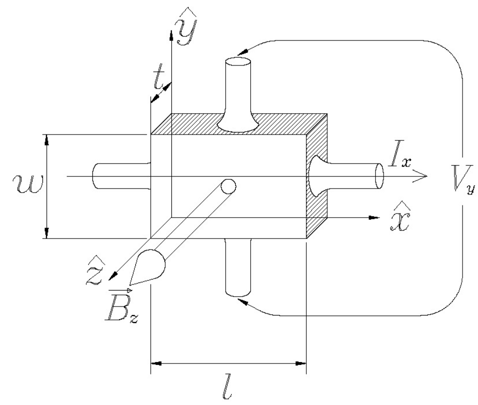
(d) Do the same for a thin wafer-shaped cavity (Fig. (c)) perpendicular to **M**. [Hint: Assume the cavities are small enough. Carving out a cavity is the same as superimposing an object of the same shape but opposite polarization.] (5%)

4. (7%,7%,6%) Consider a conducting slab as shown below with length l in the x direction, width w in the y direction and thickness t in the z direction. The conductor has charge carrier of charge q and drift velocity v_x when a current I_x flows in the positive x direction. The conductor is placed in a magnetic field perpendicular to the plane of the slab $\mathbf{B} = B_z\hat{\mathbf{z}}$.

(a) When steady state is reached, there will be no net flow of charge in the y direction. Find the relation between E_y, B_z, and v_x.

(b) Find the resulting potential difference V_y (the **Hall voltage**) **between the top and bottom of the slab**, in terms of B_z, v_x, and the relevant dimensions of the slab.

(c) How do you determine the sign of the mobile charge carriers in a material? [Hint: n denotes the number of carriers per unit volume]



5. (7%,7%,6%) The magnetic field on the axis of a circular current loop is far from uniform. We can produce a more nearly uniform field by using two such circular loops a distance d apart. This arrangement is known as a **Helmholtz coil**.

(a) Find the total magnetic field **B** along the z-axis as a function of z.

(b) Show that $\partial B/\partial z$ is zero at the point midway between them.

(c) Determine d such that $\partial B^2/\partial z^2=0$ at the midpoint, and find the resulting magnetic field at the center (z=0)